

# Quantum phase transition in an array of coupled dissipative cavities

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(Dated: December 16, 2014)

The features of superfluid-Mott insulator phase transition in the array of dissipative nonlinear cavities are analyzed. We show analytically that the coupling to the bath can be reduced to renormalizing the eigenmodes of atom-cavity system. This gives rise to a localizing effect and drives the system into mixed states. For the superfluid state, a dynamical instability will lead to a sweeping to a localized state of photons. For the Mott state, a dissipation-induced fluctuation will suppress the restoring of long-range phase coherence driven by interaction.

PACS numbers: 42.50.Pq, 42.60.Da, 64.70.Tg, 03.65.Yz

One of the remarkable applications of coupled cavity arrays is to realize quantum simulators [1–4]. Relying on the controllability of optical systems, it could be useful to attack some unclear physics and to explore new phenomenon in quantum many-body systems [5–9]. In particular, over the past years the experimental progresses in engineering strong interaction of photons and atoms [10–13] and in fabricating large-scale arrays of high-quality cavities [14, 15] make this potential application may become a reality in the near future. However, the quantum optical systems in general couple to an external environment [16, 17], which will bring the system out of equilibrium and profoundly affect the dynamics of interest [18, 19]. New important questions thus arise and need to be clarified, e.g. under the realistically experimental conditions, how the dissipation and decoherence would behave in these open systems.

In this paper, we propose a possible answer to the above question by investigating the superfluid-Mott insulator phase transition in the array of dissipative cavities. We show that the transition shares part features of the non-dissipative counterparts. There are still two quantum many-body states can be recognized as the delocalized and localized of photons. However, very differently, the dissipation and the decoherence give rise to a localizing effect and drive the system into mixed states. For the superfluid state, a non-equilibrium dynamical instability can lead to a sweeping to a localized state at a finite time. For the Mott state, where photons are already localized at each lattice site, the localization holds but a dissipation-induced fluctuation of photon number acted on each lattice site will suppress the restoring of long-range phase coherence.

Consider a system consisted of atoms and cavities coupled weakly to a bosonic environment at zero temperature. As the size of individual cavities is generally much smaller than their spacing, we assume the photons emitted from each cavity are uncorrelated. The total Hamiltonian therefore reads

$$H = H_s + H_{bath} + H_{coup}. \quad (1)$$

where  $H_s$  is the Hamiltonian for the system,  $H_{bath} =$

$\sum_j \sum_{\alpha,k} \omega_{k_\alpha} r_{j,k_\alpha}^\dagger r_{j,k_\alpha}$  the Hamiltonian for environment, and  $H_{coup} = \sum_j \sum_{\alpha,k} (\eta_{k_\alpha}^* r_{j,k_\alpha}^\dagger \alpha_j + h.c.)$  the coupled term.  $\alpha = a, c$  labels the operators and physical quantities associated with atoms and cavities, respectively.  $\omega_{k_\alpha}$  denotes the frequency of environmental modes,  $r_{j,k_\alpha}^\dagger$  and  $r_{j,k_\alpha}$  the creation and annihilation operators of quanta in the  $k_\alpha$ th mode on the  $j$ th lattice site, and  $\eta_{k_\alpha}$  the coupling strength. Here we set  $\hbar = 1$ .

The system we modeled, as depicted in Fig. 1, is a two-dimensional array of resonant optical cavities, each embedded with a two-level (artificial) atom coupled strongly to the cavity field. The possible realizations such as photonic bandgap cavities and superconducting stripline resonators et al. [4]. With  $\omega_a$  and  $\omega_c$  being the frequency of atom transition and cavity mode

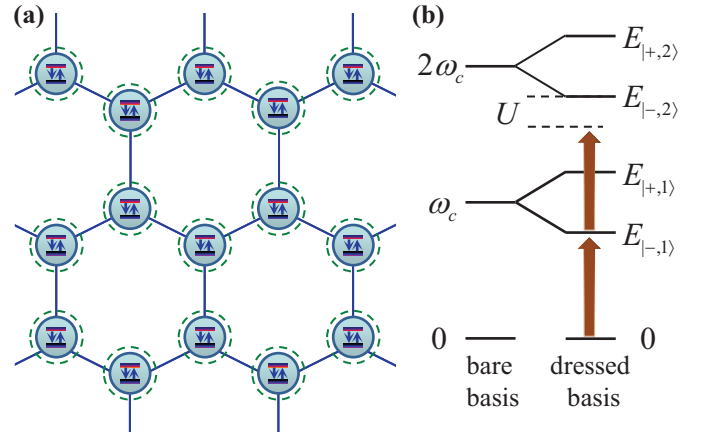


FIG. 1: A type of possible topologies for two-dimensional cavity arrays, for  $z$  nearest neighbors. (a) Individual cavities are coupled resonantly to each other due to the overlap of the evanescent fields. Each cavity contains a two-level system coupled strongly to the cavity field and immerses in a bosonic bath (marked by the dash line). (b) Energy eigenvalues of individual cavity-atom system on each site.  $\omega_c = \omega_a$  is assumed for simplicity. The anharmonicity of the Jaynes-Cummings energy levels can effectively provide an on-site repulsion  $U$  to block the absorption for the next photon.

respectively, in the rotating wave approximation(RWA), such individual atom-cavity system on site  $j$  is well described by the Jaynes-Cummings Hamiltonian,  $H_j^{JC} = \omega_c b_j^\dagger b_j + \omega_a \sigma_j^+ \sigma_j^- + \beta(\sigma_j^+ b_j + h.c.)$ . Here  $b_j^\dagger$  and  $b_j$  ( $\sigma_j^+$ ,  $\sigma_j^-$ ) are photonic(atomic pseudo-spin) raising and lowering operators, respectively,  $\beta$  the coupled strength. In the grand canonical ensemble,  $H_s$  is therefore given by combining  $H_j^{JC}$  with photonic hopping term and chemical potential term,

$$H_s = \sum_j H_j^{JC} - \sum_{\langle j,j' \rangle} \kappa_{jj'} b_j^\dagger b_{j'} - \sum_j \mu n_j. \quad (2)$$

$\kappa_{jj'}$  is the photonic hopping rate between cavities. Since the evanescent coupling between cavities decreases with the distance exponentially, we restrict the summation  $\sum_{\langle j,j' \rangle}$  running over the nearest-neighbors.  $n_j = b_j^\dagger b_j + \sigma_j^+ \sigma_j^-$  counts the total number of atomic and photonic excitations on site  $j$ .  $\mu$  is the chemical potential, where the assumption  $\mu = \mu_j$  for all sites has been made.

Due to the strong coupling, as shown in Fig. 1(b), the resonant frequencies of individual atom-cavity system are split into  $E_{|\pm, n\rangle} = n\omega_c \pm \sqrt{n\beta^2 + \frac{\Delta^2}{4}} - \frac{\Delta}{2}$ , where  $|\pm, n\rangle$  labels the positive(negative) branch of dressed states,  $\Delta = \omega_c - \omega_a$  is the detuning. The anharmonicity of the Jaynes-Cummings energy levels can effectively provide a on-site repulsion. For instance, the resonant excitation by a photon with frequency  $E_{|\pm, 1\rangle}$  will prevent the absorption of a second photon at  $E_{|\pm, 1\rangle}$ , which is the striking effect known as photon blockade [13]. It is therefore feasible to realize a quantum simulator in terms of the system described by Eq. (2). This so called Jaynes-Cummings-Hubbard(JCH) model is recently suggested by Greentree et al. [2].

However, the situation changes dramatically once taking the degrees of freedom of environment into consideration, as described by Hamiltonian(1). A non-equilibrium dynamics for open quantum many-body system do arise, which is a formidable task to solve. Here we propose a new method to eliminate those external degrees of freedom. To approach this, we regroup Hamiltonian(1) as

$$H = H_{local} - \sum_{\langle j,j' \rangle} \kappa_{jj'} b_j^\dagger b_{j'} - \sum_j \mu n_j, \quad (3)$$

where  $H_{local} = \sum_j H_j^{JC} + H_{bath} + H_{coup}$ .

First considering the case that the  $j$ th cavity contained a initial photon interacts with a bath, the dynamics is governed by

$$H_j = \omega_c b_j^\dagger b_j + \sum_k \omega_{k_c} r_{j,k_c}^\dagger r_{j,k_c} + \sum_k (\eta_{k_c}^* r_{j,k_c}^\dagger b_j + h.c.). \quad (4)$$

We denote its eigenvalue as  $\omega$  and expand the eigenvector  $|\phi_j\rangle$  as  $|\phi_j\rangle = e_c b_j^\dagger |\emptyset\rangle + \sum_k e_k r_{k_c}^\dagger |\emptyset\rangle$ .  $e_c$  and  $e_k$  are the probability amplitudes for the excitation occupied by cavity field and environment, respectively.  $|\emptyset\rangle$

denotes the vacuum state. Deducing the equations of these two amplitudes, one can express  $e_k$  in terms of  $e_c$  and integrate out the degrees of freedom of environment when the coupling to environment is weak, thus obtain  $(\omega_c + \delta\omega_c - i\gamma_c)e_c = \omega e_c$ .  $\delta\omega_c$  is known as an analog to the Lamb shift in atomic physics and is sufficiently small.  $\gamma_c$  is the decay rate and indicates a finite lifetime of cavity mode. We note that the above treatment is precise under the Born-Markov approximation [20].

This motivates us to introduce a quasi-boson described by  $B_j$  with a complex eigenfrequency  $\Omega_c = \omega_c - i\gamma_c$ , where  $\delta\omega_c$  has been absorbed into  $\omega_c$ , to redescribe the cavity field coupled with a bath in terms of  $H_j^{eff} |\phi_j\rangle = \Omega_c |\phi_j\rangle$ .  $H_j^{eff} = \Omega_c B_j^\dagger B_j$  is the effective Hamiltonian and now  $|\phi_j\rangle = e_c B_j^\dagger |\emptyset\rangle$  denotes the time-dependent damped basis [21]. Because of loss, the system would be nonconservative and corresponding operators would be non-Hermitian. The commutation relation of  $B_j$  reads  $[B_j, B_{j'}^\dagger] = (1 + i\frac{\gamma_c}{\omega_c})\delta_{jj'}$ . Recognizing  $\frac{\gamma_c}{\omega_c}$  is in order of  $\frac{1}{Q}$ , with  $Q$  being the quality factor of individual cavity. The bosonic commutation relation is therefore approximately satisfied for the high- $Q$  cavity, which can be met in most experiments about cavity quantum electrodynamics(QED).

The complex eigenfrequency underlines the facts that, on one hand, dissipation is the inherent property for realistic cavity. When a photon with certain frequency has been injected into a dissipative cavity, the composite system can not be characterized only by the mode of cavity field, however, we must take the impacts of environment into account. On the other hand, in general we do not concern the time evolution of bath. In this way, the array of dissipative cavities can be regarded as a configuration consisted of quasi-bosons. Quite similar operations can be performed on atom to introduce another kind of quasinormal mode described by  $\tilde{\sigma}_j^\pm$  with the frequency  $\Omega_a = \omega_a - i\gamma_a$ , where  $\gamma_a$  is the atomic decay rate.

We can therefore rephrase Hamiltonian (3) with the renormalized terms,

$$H = \sum_j H_j^{eff} - \sum_{\langle j,j' \rangle} \kappa_{jj'} B_j^\dagger B_{j'} - \sum_j \mu n_j, \quad (5)$$

with now  $H_j^{eff} = \Omega_c B_j^\dagger B_j + \Omega_a \tilde{\sigma}_j^+ \tilde{\sigma}_j^- + \beta(\tilde{\sigma}_j^+ B_j + h.c.)$  and  $n_j = B_j^\dagger B_j + \tilde{\sigma}_j^+ \tilde{\sigma}_j^-$ . One nice feature of Hamiltonian (5) is now the losses describe by leaky rates  $\gamma_a$  and  $\gamma_c$  but not by operators. Without having to mention the external degrees of freedom, this effective treatment would be of great conceptual and, moreover, computational advantage rather than the general treatment as Hamiltonian (1). A more microscopic consideration points out that, in cavity QED region, since the atom is dressed by cavity field, the atom and field act as a whole subject to a total decay rate  $\Gamma$  [22]. In particular,  $\Gamma = n(\gamma_a + \gamma_c)$  for  $\Delta = 0$ .

To gain insight over the role of dissipation in the superfluid-Mott insulator phase transition, we use a mean

field approximation which could give reliable results comparing to the Monte Carlo calculations if the system is at least two-dimensional [23]. We introduce a superfluid parameter,  $\psi = \text{Re}\langle B_j \rangle = \text{Re}\langle B_j^\dagger \rangle$ . In the present case, the expected value of  $B_j(B_j^\dagger)$  is in general complex with the formation  $\langle B_j \rangle = \psi - i\psi_\gamma$  ( $\langle B_j^\dagger \rangle = \psi + i\psi_\gamma$ ).  $\psi_\gamma$  is a solvable small quantity as a function of  $\gamma_a$  and  $\gamma_c$ , and vanishes in the limit of no loss. Using the decoupling approximation,  $B_j^\dagger B_{j'} = \langle B_j^\dagger \rangle B_{j'} + \langle B_{j'} \rangle B_j^\dagger - \langle B_j^\dagger \rangle \langle B_{j'} \rangle$ , the resulting mean-field Hamiltonian can be written as a sum over single sites,

$$H^{MF} = \sum_j \{ H_j^{eff} - z\kappa\psi(B_j^\dagger + B_j) + z\kappa|\psi|^2 - \mu n_j + O(\psi_\gamma^2) \}, \quad (6)$$

where we have set the intercavity hopping rate  $\kappa_{jj'} = \kappa$  for all nearest-neighbors with  $z$  labeling the number. For zero temperature, this mean-field approximation is equivalent to the Gutzwiller approximation, which assumes the wave function of system as a product of single-site wave function [24].

$\psi$  can be examined analytically in terms of the second-order perturbation theory, with respect to the damped dressed basis. For energetically favorable we assume each site is prepared in the negative branch of dressed state. But because the dressed basis is defined on  $n \geq 1$ , a ground state  $|0\rangle$  with the energy  $E_{|0\rangle} = 0$  need to be supplemented. Thus

$$\psi = e^{-\Gamma t} \sqrt{-\frac{\chi}{z\kappa\Theta}}. \quad (7)$$

$\chi$  and  $\Theta$  are functions of all the parameters of the whole system. Since the evanescent parameter  $\kappa$  is a typical small quantity in systems of coupled cavities, the perturbation theory gives good qualitative and even quantitative descriptions comparing to the numerically results given by explicitly diagonalizing [2, 25].

Arguably the most interesting situation is the effective photon-photon interactions are maximized, namely, cavities on resonant with atoms and with one initial excitations per lattice site [26]. And thereby  $\Gamma = \gamma_a + \gamma_c = \gamma$ . With  $F_1 = \omega_c - \beta - \mu$  and  $F_2 = -\omega_c + (\sqrt{2} - 1)\beta + \mu$ , in eq. (7),  $\Theta = \frac{1}{2F_1^2 + 2\gamma^2} + \frac{3+2\sqrt{2}}{4F_2^2 + 4\gamma^2} > 0$ , and

$$\chi = \frac{F_1}{2F_1^2 + 2\gamma^2} + \frac{(3+2\sqrt{2})F_2}{4F_2^2 + 4\gamma^2} + \frac{1}{z\kappa e^{-2\gamma t}}. \quad (8)$$

In the absence of loss, one can recognize  $\chi = 0$  is the well known self-consistent equation and therefore distinguish the superfluid phase and Mott phase. Nevertheless, the coupling to environment inducing a non-equilibrium dynamics, thus no strict phase exists. However, provided the external time dependence is much slower than the internal frequencies of system, there remains two fundamentally different quantum state can be identified through whether  $\psi$  vanishes or has a finite value, i.e. photons localized in each lattice site and delocalized across the cavities.

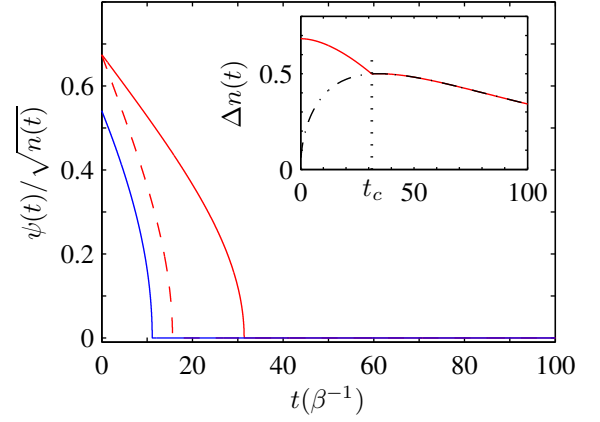


FIG. 2: The time dependence of the long-range phase coherence and the photon number fluctuation on each site for a certain initial state (inset). For a initial superfluid state (the red line for  $\frac{z\kappa}{\beta} = 0.3$  and blue line for  $\frac{z\kappa}{\beta} = 0.2$ ), before  $t_c$  the long-range order decays continuously and the fluctuation is a total effect of photon hopping and photon leakage (the solid line for  $\frac{\gamma}{\beta} = 0.01$  and dash line for  $\frac{\gamma}{\beta} = 0.02$ ). Beyond  $t_c$ , the superfluidity breaks down and the fluctuation behaves as the localized state (the dot-dash line for  $\frac{z\kappa}{\beta} = 0$  and  $\frac{\gamma}{\beta} = 0.01$ ).

To analyze the physics of the transition between these two states in detail, we proceed our discussion from two aspects. First, we start with the superfluid phase and track the time evolution of long-range phase coherence. The prefactor  $e^{-\Gamma t}$  in Eq. (7) indicates the expected decay of  $\psi$ . However, more importantly, a dynamical instability due to the coupling to the external environment is revealed by  $\chi$ . As illustrated in Fig. 2, for  $t \ll \beta^{-1}$ ,  $\psi$  has a slightly reduction scaled by  $\frac{\gamma^2}{\beta^2}$ . For  $t > \beta^{-1}$ ,  $z\kappa e^{-2\gamma t}$  is the leading term and pronounces the decrease of effective tunneling energy. Consequently, a photon hopping rate  $\kappa$  given initially in the superfluid region will cross the critical point at a time  $t_c \simeq \frac{1}{2\gamma} \ln \frac{\kappa}{\kappa_c}$ , with  $\kappa_c$  being the critical tunneling energy for a given  $z$  and  $\beta$ . Before  $t_c$ , a non-local region is still recognized as non-local. The dissipation has not changed the fundamental nature of the system, albeit with the reduction of long-range phase coherence and an additional fluctuation due to photon leakage. Nevertheless, beyond  $t_c$ , the superfluidity breaks down, i.e. a transition to the localized state do occur. An analogous localizing effect is described in an optical lattice system very recently, where the spontaneous emission of atoms owing to the lattice heat leads to decoherence of many-body state [27].

In what follows, in contrast, we start in the Mott state and discuss the impacts of dissipation on the critical behavior and the fluctuation behavior. Consider the initial state is deep in the Mott phase,  $\frac{z\kappa}{\beta} = 0$ , and we continuously increase the intercavity coupled rate. For the related ideal case, one can reach the superfluid phase at  $\frac{z\kappa}{\beta} = (\frac{z\kappa}{\beta})'_c \simeq 0.16$ . However, the presence of bath converts coherences originally in the system into entangle-

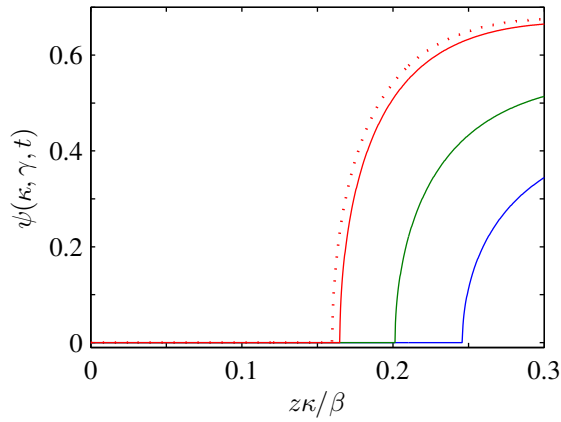


FIG. 3: The restoring of long-range phase coherence from the Mott state. Influences of dissipation depend on the leaky rate  $\gamma$  (the dot and solid line for  $\frac{\gamma}{\beta} = 0$  and  $0.05$ , respectively) and will accumulate along with time (the red, green, and blue lines for  $t = 0, 0.1\gamma^{-1}$ , and  $0.2\gamma^{-1}$ , respectively).

ment of the system and the environment [11], thus the effective tunneling energy will be lower than expected. Moreover, this impact will accumulate along with time. As shown in Fig. 3, to expect the appearance of photonic hopping we must keep increasing  $\kappa$ . On the other hand, despite the long-range order is still absent, different from the pure Mott state, there will be a fluctuation owing to photon leakage acted on each lattice site (dot-dash line in Fig. 2). Consequently, we will not be able to restore the long-range phase coherence perfectly by driven  $\frac{z\kappa}{\beta}$  into the superfluid region.

In summary, we have shown analytically the features of superfluid-Mott insulator phase transition in the array of dissipative cavities. Our analysis sufficiently takes into account the intrinsically dissipative nature of open quantum many-body system, and identifies how dissipation and decoherence would come into play. For the further experimental signature, we predict that there will be a localizing effect.

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